Inspiring Greatness

ISD

INDEPENDENCE SCHOOL DISTRICT

Math Virtual Learning

Calculus AB

Volume: The Disk and Washer Methods

April 24, 2020

Calculus AB
Lesson: April 24, 2020

Objective/Learning Target:
Students will calculate the volume of a solid of revolution using the disk and washer methods.

Warm-Up:

Note: This is a review of the previous two days lessons so please refer back to those lessons (April 22 and April 23) as well.

Watch Video: More Washer Method

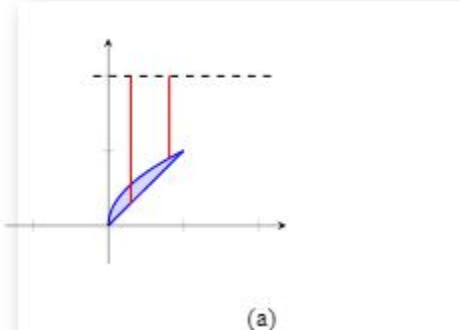
Read Article: Review Disk and Washer Method

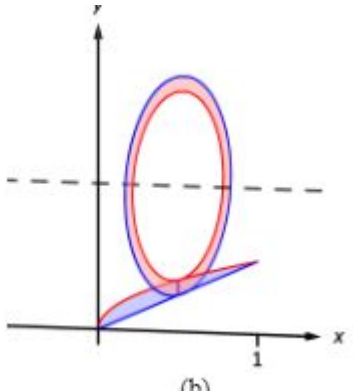
Examples:

Find the volume of the solid formed by rotating the region bounded by $y = \sqrt{x}$ and $y = x$ about $y = 2$.

Solution Figure 6.2.9 shows the region we are rotating (a), a sample slice (b) and the full solid (c). The axis of rotation is horizontal so the radii must be functions of x. The radii is the distance from the axis of rotation to the curve so the outside radius of this washer is $R(x) = 2 - x$ and the inside radius is $r(x) = 2 - \sqrt{x}$. The region is bounded from $x = 0$ to $x = 1$, thus the volume is

$$\begin{aligned} V &= \pi \int_0^1 \left((2-x)^2 - (2-\sqrt{x})^2 \right) \, dx \\ &= \pi \int_0^1 (4 - 4x + x^2) - (4 - 4\sqrt{x} + x) \, dx \\ &= \pi \int_0^1 x^2 - 5x + 4\sqrt{x} \, dx \\ &= \pi \left[\frac{1}{3}x^3 - \frac{5}{2}x^2 + \frac{8}{3}x^{3/2} \right] \Big|_0^1 \\ &= \frac{\pi}{2} \text{units}^3. \end{aligned}$$





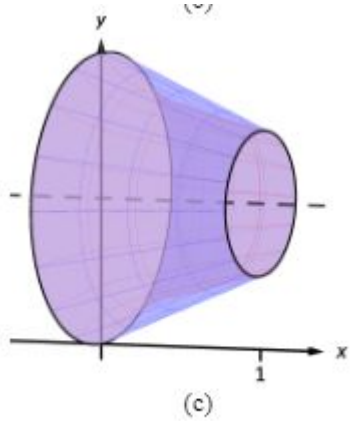


Figure 6.2.9: Sketching the solid

Examples:

Find the volume of the solid generated when the area bounded by the curve $y^2 = x$, the x-axis and the line $x = 2$ is revolved about the x-axis.

Solution: Circular Disk Method

▾ Click here to show or hide the solution

$$V = \pi \int_{x_1}^{x_2} {y_U}^2 \, dx$$

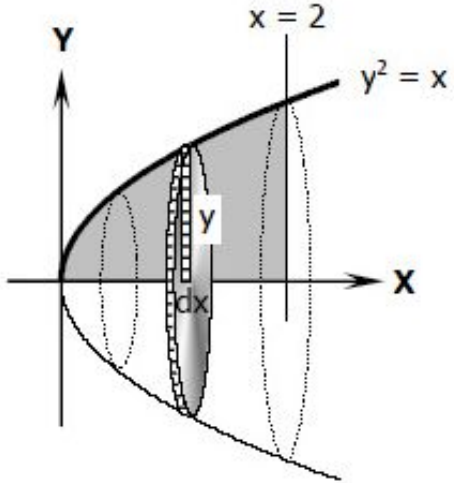
$$V = \pi \int_0^2 (x^{1/2})^2 \, dx$$

$$V = \pi \int_0^2 x \, dx$$

$$V = \pi \left[\frac{x^2}{2} \right]_0^2$$

$$V = \tfrac{1}{2}\pi [2^2 - 0^2]$$

$$V = 2\pi \text{ unit}^3 \qquad \textit{answer}$$



Practice:

Find the volume of the solid obtained by rotating the region bounded by the given curves about the given axis. Use the method of slicing (disks/washers). Sketch the region and a typical disk or washer.

1) $y = x^2 - 2x$, $y = 8$; about the line $y = 8$

2) $y = x^2$, $x = 1$, $y = 0$; about the line $x = 1$

Answer Key:

Once you have completed the problems, check your answers here.

1) $\frac{1296\pi}{5}$

2) $\frac{\pi}{6}$

Additional Practice:

Interactive Practice

Extra Practice with Answers